BU CLA CS 530: Analysis of Algorithms

Spring 1996

Problem Set 12

Due Thursday 96.04.18, at 11:00 am, in class. Late submissions not accepted.

---

1. Exercise 36.2-8, page 929, in [CLR].

2. In [CLR], page 950, it is shown that VERTEX-COVER is NP-hard by the reduction CLIQUE $\leq_P$ VERTEX-COVER. Prove the converse reduction, namely VERTEX-COVER $\leq_P$ CLIQUE. *Hint*: Let the proof on page 950 inspire you.

3. Let $G = (V, E)$ be an undirected graph. A set of vertices $V' \subseteq V$ is called a *dominating set* for G if every vertex is either in V' or connected by some edge to a vertex in V'. The problem DOMINATING-SET is, given an undirected graph G and a positive integer k, to determine whether there is a dominating set for G with k vertices. Show that DOMINATING-SET is NP-complete.
   Hint: Use the reduction VERTEX-COVER $\leq_P$ DOMINATING-SET to show the NP-hardness part. Consider Figure 1 for further inspiration.

4. Let k be a fixed positive integer. Consider a restricted version of SAT, called SAT_k, where formulas may contain at most k occurrences of any variable, negated or unnegated. (The definition of SAT is given in [CLR], page 940.)

   (a) Show that for $k \geqslant 3$, SAT_k is NP-complete.
   Hint: Show there is a reduction SAT $\leq_P$ SAT_k. Given a boolean formula φ where variable x occurs $m > 3$ times, find a new formula φ' which is satisfiable iff φ is satisfiable and where the occurrences of x are replaced by m distinct fresh variables $x_1, x_2, \ldots, x_m$. Repeat the process for every variable in the original φ.

   (b) Show that SAT_2 is solvable in polynomial time. Restrict attention to the case when the instances of SAT_2 are boolean formulas in CNF.
   Hint: $(x_1 \vee y_1 \vee y_2) \wedge (\neg x_1 \vee z_1 \vee z_2 \vee z_3)$ is satisfiable iff $(y_1 \vee y_2 \vee z_1 \vee z_2 \vee z_3)$ is satisfiable.

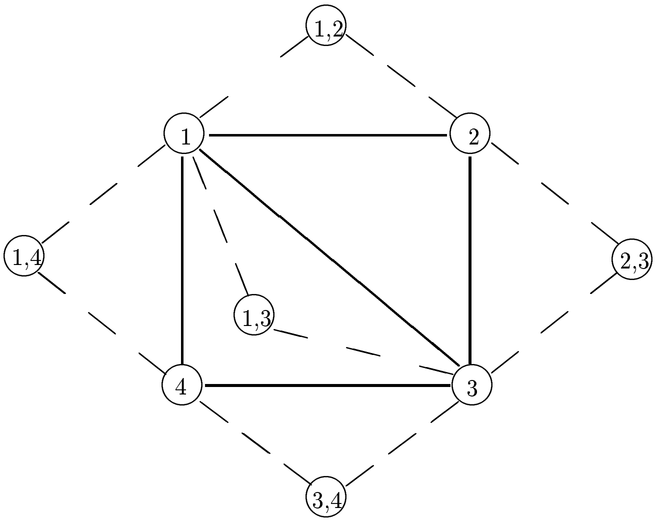

Figure 1. The reduction for Problem 2.
The original vertex set is $V = \{1, 2, 3, 4\}$.
The new vertex set is $V' = V \cup \{(1,2), (1,3), (1,4), (2,3), (3,4)\}$.

Created: 96.04.11