BU CLA CS 530: Analysis of Algorithms

Spring 1996

Problem Set 2

Due Thursday 96.02.01, at 11:00 am, in class. Late submissions not accepted.

---

1. Prove by induction on $n \geqslant 0$ that $\sum_{i=0}^{n} F_i = F_{n+2} - 1$, where F_n is the n-th Fibonacci number (definition in [CLR], page 36). Show clearly the steps of your inductive proof.

2. What is the function computed by the algorithm FUN, where n is an integer $\geqslant 0$:

```
FUN(n)
1.    if n ⩽ 1
2.    then return n
3.    else return 5 · FUN(n − 1) − 6 · FUN(n − 2)
```

   Hint: The function is $3^n - 2^n$. But you have to prove it!

3. Analyze the recursive algorithm FUN: How many times the subtraction operation in line 3 is executed in the worst-case? (We only count the operation once when we subtract $6 \cdot \text{FUN}(n-2)$ from $5 \cdot \text{FUN}(n-1)$, i.e. we do not count as subtractions the two decrements "$n-1$" and "$n-2$".) *Hint*: Solve an appropriately defined recurrence by iteration, and observe a pattern involving Fibonacci numbers. Use the result of Problem 1 above. Use the rate of growth of Fibonacci numbers given at the top of page 37, in [CLR], to obtain an asymptotic running time.

4. Exercise 9.2-3, page 177, in [CLR]. Justify your answer.

5. Exercise 10.1-1, page 187, in [CLR].

6. Exercise 10.2-2, page 189, in [CLR].

Created: 96.01.25