BU CLA CS 530: Analysis of Algorithms

Spring 1999

Problem Set 14

Due Tuesday May 4 (not Thursday April 29) by 5:00 pm, in my mailbox in the main office of the CS department. Late submissions not accepted.

An input instance of the MAX-k-BIN-PACK problem, where $k \geqslant 1$, is the same as an input instance of the BIN-PACK problem, i.e. a set $S = \{s_1, \ldots, s_n\}$ of positive integers together with a bin capacity B which is also a positive integer. The desired output is the largest number m of objects in S that can be packed into k bins.

1. Show that MAX-2-BIN-PACK is NP-hard.

We say that an approximation algorithm for a minimization problem (resp., maximization problem) has an *absolute error* $e > 0$ if for any input of size n, the cost C of the solution returned by the approximation algorithm is such that $C^* \leqslant C \leqslant C^* + e$ (resp., $C^* - e \leqslant C \leqslant C^*$). In lecture I used the expression "constant-error approximation algorithm" instead of "absolute-error approximation algorithm", but the latter expression seems more appropriate.

2. Design a polynomial-time approximation algorithm with absolute error $e = 1$ for the MAX-2-BIN-PACK problem.

3. Let e be an arbitrary positive integer. Show that if P $\neq$ NP, then there is no polynomial-time approximation algorithm for VERTEX-COVER which has absolute error e.

The definition of a *fully polynomial-time approximation scheme* is given at the bottom of page 965 and the top of page 966 in [CLR].

4. Show that if P $\neq$ NP, then there is no fully polynomial-time approximation scheme for BIN-PACK.

5. Problem 37-2 part ***a*** only, page 984, in [CLR]. Do the case $k = 2$ only.
   Hint: Try the construction on a small example, e.g. a graph with 3 or 4 vertices.

6. Show that if (the optimization version of) the clique problem has a polynomial-time approximation algorithm with relative error ε for *some* $0 < \varepsilon < 1$, then for *every* $0 < \varepsilon' < 1$ the clique problem has a polynomial-time approximation algorithm with relative error ε'.
   Hint: Do the preceding problem first. Then, based on the presumed approximation algorithm with relative error ε, construct another approximation algorithm with relative error δ, for some $\delta < \varepsilon$. Generalize for arbitrary $\varepsilon' < \varepsilon$.

Created: 99.04.22
Modified: 99.04.26