BU CLA CS 530: Analysis of Algorithms

Spring 1999

Problem Set 7

Due Thursday March 4, by 5:00 pm, in my mailbox in the main office of the CS department. Late submissions not accepted.

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1. (a) Exercise 6.2-3, page 109, in [CLR].
   (b) Three numbers are selected uniformly at random from the set $S = \{1, \ldots, 2n\}$ where $n \geqslant 1$. What is the probability that the sum of the three numbers is even? *Hint*: Note that S contains an equal number of even and odd numbers, which should make your calculation a little easier.

2. Solve the recurrence: $f(n) = \lceil \sqrt{n} \rceil f(\lceil \sqrt{n} \rceil) + c \cdot n$, where n ranges over the natural numbers and c is a positive constant. *Hint*: First, let $g(n) = f(n)/n$ and transform the recurrence accordingly. Then, let $h(n) = g \cdots$, transform the recurrence once more, and solve.

3. In general it is sufficient to evaluate recurrences only for powers of 2 (or 3 or some other constant), but this is not always the case. Consider the following recurrence:

$$T(n) = \begin{cases} T(n/2) + 1, & \text{if } n \text{ even,} \\ 2 \cdot T((n-1)/2), & \text{if } n \text{ odd,} \end{cases}$$

   with $T(1) = 1$.

   (a) Show that the solution of this recurrence for powers of 2 is $T(2^k) = 1 + k$, i.e. for all n that are powers of 2, $T(n) = 1 + \log n = \mathcal{O}(\log n)$.
   (b) Show that, for an infinite number of values of n, $T(n) = (n+1)/2 = \Omega(n)$.
   (c) State a condition on a recurrence so that the usual restriction to powers of 2 (or some other constant) is sufficient, in order to correctly determine the asymptotic behavior of the solution for all n.

4. Exercise 27.3-3, page 604, in [CLR].

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