BU CLA CS 530: Analysis of Algorithms
Spring 1999

Selected Solutions for Problem Set 2

Exercise 10.2-2

The worst case partition occurs when always partitioning around the maximum element in the array. In this case it would partition around 9, then 8, ... then finally 1.

Exercise 10.3-9

Let Y be the distance of the East-West pipeline from the x-axis. We want to choose Y so that the distance D defined by

$$D = |Y - y_1| + |Y - y_2| + \ldots + |Y - y_n| \tag{1}$$

is minimized. A moment of thought shows that D is minimized whenever "the number of wells North of the pipline" is queal to the "number of wells South of the pipeline". It suffices therefore to set Y to the value of the median in the sequence $< y_1, \ldots, y_n >$. If the sequence has two medians, Y can be set to either of these two values.

Problem 10-1

Part a

Using a comparison based algorithm, the optimum time we could have would be $\theta(n\log(n))$. Finding the first i numbers in a sorted array takes $\theta(i)$ time. Since $i \leq n$ the final time is $\theta(n\log(n))$.

Part b

Building a priority queue of n numbers takes $\theta(n)$ time using BUILD-HEAP. Each call to HEAP-EXTRACT-MAX takes $\theta(\log n)$ time. The total running time is $\theta(n + i\log(n))$.

Exercise 2 from Handout 2

Adapting the UPPER-HALF algorithm from the handout we get,

UPPER-FIFTH(A)

1. n ←length(A)
2. if (n < 389)
3. then
4. return MAXIMUM(A[1 . . . $\left(\frac{4n}{5}\right)$])
5. else
6. for i ←1 to 311
7. do
8. B[i] ←RANDOM(1,n)
9. return MAXIMUM(B[1 . . . 311])

Exercise 8 from Handout 8

Adapting POLY-INDENTITY from the handout we get,

POLY-EQUAL(f,g)

1. n ←number of variables in f
2. m ←number of variables in g
3. k ←degree of f
4. j ←degree of g
5. if (n $\neq$ m) and (k $\neq$ j)
6. return "f is not identically equal to g"
7. l ←0
8. repeat
9. for i ←1 to n do a[i] ←RANDOM(1,2kn)
10. b ←f(a[1]. . . a[n])
11. c ←g(a[1]. . . a[n])
12. if ($b - c \neq 0$)
13. return "f is not identically equal to g"

14. l ←l+1

15. until l = 100

16. return "f is identically equal to g"

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Created: 99.02.07
Modified: 99.02.16