BU CLA CS 530: Analysis of Algorithms
Spring 1999

Selected Solutions for Problem Set 3

Exercise 16.3-5

Using the given hint, we first sort the original array using mergesort $(O(nlg(n)))$. We then could run LCS-Length to get the logest coomen subsequence, which runs in a maximum of $O(n^2)$ time.

Exercise 16.3-6

(a) We can fill the array in $O(nlog(n))$ time. Since each row is a copy of the previous row with the exception of one switch, copying the row takes little time. Performing the switch can be done with a $O(log(n))$ binary search algorithm. Since there are n rows the total operation is $O(nlog(n))$.

(b) Building C will also take $O(nlog(n))$ time. Again we could use a divide and conquor which takes $O(log(n))$ time to find the largest non-infinity element. We need to do this on n rows, so this part runs in time $O(nlog(n))$.

(c) Building D will take $O(n)$ time. We could run an algorithm which does the same task as Print-LCS which will run in $O(n)$ time.

Exercise 16.4-2

The professor is right in his comment. Given the fact that the area of a polygon will remain constant for any configuration of chords, than any configuration will be optimal. Any arbitrary configuration is optimal.

Exercise 31.2-3

The recurrence

$$T(n) = kT\left(\frac{n}{3}\right) + O\left(n^2\right)$$

Using the master's theorem to solve the recurrence we obtain $n^{log_3 k}$. We are looking for

$$\begin{aligned} n^{log_3 k} &< n^{log_2 7} \\ k &= 21 \end{aligned}$$

Exercise 31.5-1

We need to prove that M(n) ¡ O(S(n)) and also S(n) ¡ O(M(n)). We could prove that squaring a matrix is bounded by the time it takes to multiply two matrices because squaring is the same as multiplication where both multiplicands are the same.

To prove that matrix multiplication is bounded by the same order of magnitude as squaring, we could create a matrix D of size $2n \times 2n$ such that

$$D = \begin{pmatrix} A & B \\ 0 & 0 \end{pmatrix}$$

By squaring this matrix we would obtain

$$D^2 = \begin{pmatrix} A^2 & AB \\ 0 & 0 \end{pmatrix}$$

Thus, we could obtain multiplication in the same order as squaring.

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