CS520, Handout 23, 21 November 2006

Subtyping

Reading: [Pierce, Chapter 15]

Examples

Is it safe to evaluate the following code?

$$(\texttt{lambda}\ \texttt{x} : \texttt{Real}.\ \texttt{sqRoot}\ \texttt{x})\ 5$$

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Or the following code?

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How about the following?

$$(\texttt{lambda}\ \texttt{x} :\ \texttt{Int}.\ \texttt{prime}\ \texttt{x})\ ((\texttt{lambda}\ \texttt{x} :\ \texttt{Real}.\ \texttt{x})\ 5.3)$$

Examples

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Or the following?

$$(\texttt{lambda}\ \texttt{r} :\ \{\texttt{x} : \texttt{Nat}, \texttt{y} : \texttt{Bool}\}.\ \texttt{r.x})\ \{\texttt{x} = 0, \texttt{y} = \texttt{true}, \texttt{z} = 1\}$$

Examples

Is it safe to evaluate the following code?

$$(\texttt{lambda}\ \texttt{x} :\ \texttt{Real}.\ \texttt{sqRoot}\ \texttt{x})\ 5$$

Or the following code?

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How about the following?

$$(\texttt{lambda}\ \texttt{x} :\ \texttt{Int}.\ \texttt{prime}\ \texttt{x})\ ((\texttt{lambda}\ \texttt{x} :\ \texttt{Real}.\ \texttt{x})\ 5.3)$$

Or the following?

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Or the following?

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Examples

Is it safe to evaluate the following code? YES

$$(\texttt{lambda x : Real. sqRoot x})\ 5$$

Or the following code? YES

$$(\texttt{lambda x : Real. sqRoot x})\ ((\texttt{lambda x : Int. x})\ 5)$$

How about the following? NO

$$(\texttt{lambda x : Int. prime x})\ ((\texttt{lambda x : Real. x})\ 5.3)$$

Or the following? YES

$$(\texttt{lambda r : }\{\texttt{x}:\texttt{Nat},\texttt{y}:\texttt{Bool}\}.\ \texttt{r.x})\ \{\texttt{x}=0,\texttt{y}=\texttt{true},\texttt{z}=1\}$$

Or the following? NO

$$(\texttt{lambda r : }\{\texttt{x}:\texttt{Nat},\texttt{y}:\texttt{Bool},\texttt{z}:\texttt{Nat}\}.\ \texttt{r.x})\ \{\texttt{x}=0,\texttt{y}=\texttt{true}\}$$

None of the preceding examples is typable in a system of simple types. In order to type examples (1), (2) and (4), which are safe to evaluate, we can add the following typing rules:

$$\frac{}{\texttt{Int} <: \texttt{Real}} \qquad \text{(S-IntReal)}$$

$$\frac{}{\{\ell_i : T_i{}^{i\in 1..n+k}\} <: \{\ell_i : T_i{}^{i\in 1..n}\}} \qquad \text{(S-RcdWidth)}$$

$$\frac{\Gamma \vdash t : T \qquad T <: T'}{\Gamma \vdash t : T'} \qquad \text{(T-Sub)}$$

Only the third rule is used to assign a new type to a term t. The first two rules are used to determine whether a type is a subtype of another.

Typing Derivation For $(\texttt{lambda x : Real. sqRoot x})\ 5$

Let $\Gamma = (\texttt{x} : \texttt{Real})$:

$$
\dfrac{
\dfrac{
\dfrac{}{\Gamma \vdash \texttt{sqRoot} : \texttt{Real} \to \texttt{Real}}
\qquad
\dfrac{(\texttt{x} : \texttt{Real}) \in \Gamma}{\Gamma \vdash \texttt{x} : \texttt{Real}}
}{
\dfrac{\Gamma \vdash \texttt{sqRoot x} : \texttt{Real}}{\vdash (\lambda \texttt{x} : \texttt{Real}.\,\texttt{sqRoot x}) : \texttt{Real} \to \texttt{Real}}
}
\qquad
\dfrac{\dfrac{}{\vdash 5 : \texttt{Int}} \qquad \dfrac{}{\texttt{Int} <: \texttt{Real}}\ \textsc{S-IntReal}}{\vdash 5 : \texttt{Real}}\ \textsc{T-Sub}
}{
\vdash (\lambda \texttt{x} : \texttt{Real}.\,\texttt{sqRoot x})\ 5 : \texttt{Real}
}
$$