BU CLA MA 531: Computability and Logic

Fall 1995

Problem Set 13 (due Friday 95.12.15)

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1. Prove the *subformula property* for derivations in normal form, i.e. prove that in a normal derivation $\mathcal{D}$ for $\Gamma \vdash \varphi$, every wff occurring in $\mathcal{D}$ is a subformula of φ or of a hypothesis in Γ. (φ is also a subformula of itself.)

2. Using the subformula property of normal derivations, show that *first-order logic is conservative over propositional logic*, i.e. the additional rules for first-order logic (the rules $\forall$`I` and $\forall$`E`) do not derive propositional wff's that are not already derivable using only the rules for propositional logic.[1]

3. Show that for every propositional wff φ, we have $\vdash^C \neg\varphi \Leftrightarrow \vdash^I \neg\varphi$. (It will help to review Glivenko's theorem, started in lecture and completed in the preceding Problem Set.)

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[1] We deal with the classical case only, so that we can ignore the rules $\exists$`I` and $\exists$`E`. As in the normalization proofs in lecture, we also ignore the rules for equality, which introduce some complications we did not have time to consider.