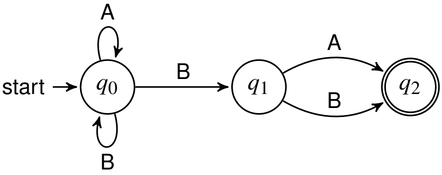
CS 512, Spring 2018, Handout 02

Finite Automata and Büchi Automata

Assaf Kfoury

23 January 2018

both kinds are defined in the same way: **example 1**



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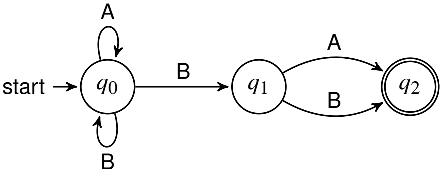

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alphabet $\Sigma = \{A, B\}$

subset of *initial* states $Q_0 = \{q_0\}$

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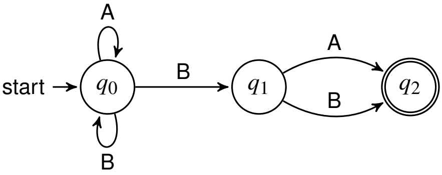

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- as nondeterministic finite automaton (NFA), call it $\mathscr{A}_1$,
  $\mathscr{A}_1$ accepts the language:

$$\mathscr{L}(\mathscr{A}_1) = \left\{ x_1 \cdots x_n B y \;\middle|\; n \geqslant 0,\ x_i \in \{A, B\},\ y \in \{A, B\} \right\}$$

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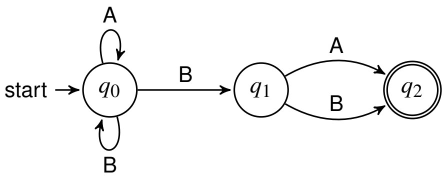

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- as nondeterministic Büchi automaton (NBA), call it $\mathscr{A}_{1,B}$,
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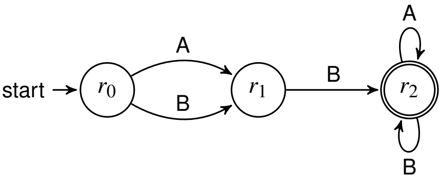
$$\mathscr{L}(\mathscr{A}_{1,B}) = \varnothing$$

both kinds are defined in the same way: **example 2**



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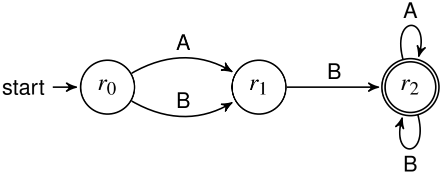

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- as nondeterministic finite automaton (NFA), call it $\mathscr{A}_2$,
  $\mathscr{A}_2$ accepts the language:

$$\mathscr{L}(\mathscr{A}_2) = \left\{ y B x_1 \cdots x_n \;\middle|\; n \geqslant 0,\ x_i \in \{A, B\},\ y \in \{A, B\} \right\}$$

also defined by the *regular expression* $(A + B) B (A + B)^*$.

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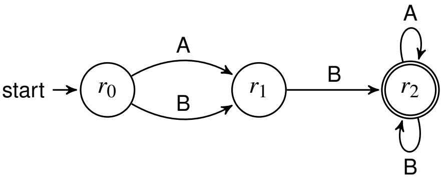

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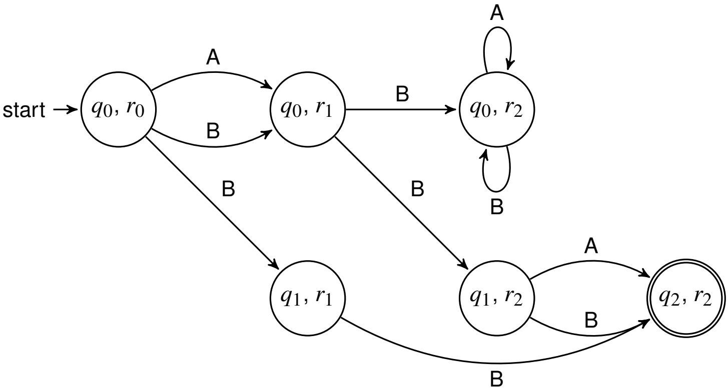
- as nondeterministic Büchi automaton (NBA), call it $\mathscr{A}_{2,B}$,
  $\mathscr{A}_{2,B}$ accepts the ω-language:

  $$\mathscr{L}(\mathscr{A}_{2,B}) = \left\{ yBx_0\, x_1 \cdots x_i \cdots \,\middle|\, i \in \mathbb{N},\ x_i \in \{A, B\},\ y \in \{A, B\} \right\}$$

  also defined by the ω*-regular expression* $(A+B)\,B\,(A+B)^\omega$.

synchronous product: **example 1+example 2** (continued)

the synchronous product of $\mathscr{A}_1$ and $\mathscr{A}_2$ (and also $\mathscr{A}_{1,B}$ and $\mathscr{A}_{2,B}$) looks as follows:



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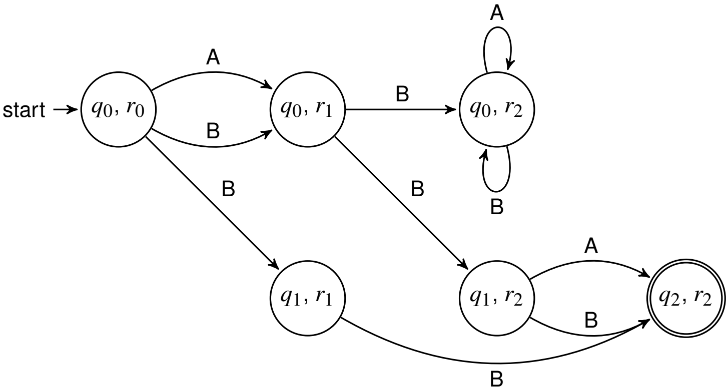
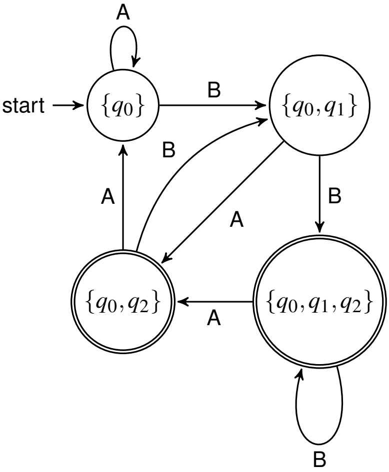


- **Fact**: $\mathscr{L}(\mathscr{A}_1) \cap \mathscr{L}(\mathscr{A}_2) = \varnothing$ if and only if $\mathscr{L}(\mathscr{A}_1 \otimes \mathscr{A}_2) = \varnothing$.

powerset construction: **example 1** (continued)

the powserset construction applied to NFA $\mathscr{A}_1$ produces
the deterministic finite automaton (DFA) below, call it $\mathscr{A}_3$:



set of states $Q =$
$\big\{\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_1, q_2\}\big\}$

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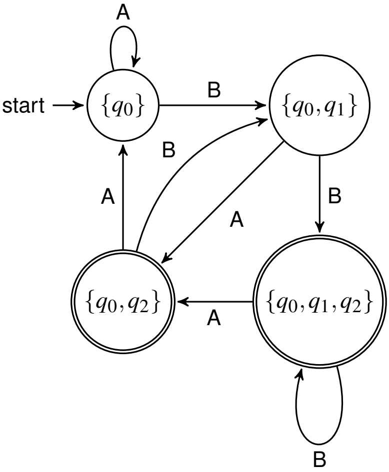

set of states $Q = \big\{\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_1, q_2\}\big\}$

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- we can view above as a deterministic Büchi automaton (DBA), call it $\mathscr{A}_{3,B}$
- however, while $\mathscr{A}_1 \equiv \mathscr{A}_3$, we have $\mathscr{A}_{1,B} \not\equiv \mathscr{A}_{3,B}$

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