# CS 512, Spring 2018, Handout 05

# $\omega$-Regular Properties

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# $\omega$-regular properties

- LT property $P$ over AP is an **$\omega$-regular property** if $P$ is an $\omega$-regular language over the alphabet $2^{\text{AP}}$. [PMC, Definition 4.25, page 172]
- LT properties that are $\omega$-regular are particularly expressive, in that they subsume the expressive power of several other LT properties, as indicated by the following implications (and in each case the reverse implication does not hold):
  1. Every *regular safety* property over AP is an $\omega$-regular property over AP. [PMC, second paragraph of page 173] and next slide.
  2. Every *invariant* property over AP is an $\omega$-regular property over AP. [PMC, top of page 173].
  3. Many *liveness* properties over AP are $\omega$-regular properties over AP.
  4. Looking ahead: Every *LTL-formula* over AP expresses an $\omega$-regular property over AP. [PMC, Remark 5.43, page 286].

More on the preceding in the handout "*Properties of Transition Systems*" click here and in the book [PMC].

# every regular safety property is an $\omega$-regular property

- If $P$ is a regular safety property over AP, then its complement is of the form:

$$\underbrace{(2^{\mathsf{AP}})^{\omega} - P}_{\text{complement of } P} \;=\; \underbrace{\underbrace{\mathsf{MinBadPref}(P)}_{\text{regular}} \cdot (2^{\mathsf{AP}})^{\omega}}_{\omega\text{-regular}}$$

- Hence, the set $(2^{\mathsf{AP}})^{\omega} - P$ of all the bad $\omega$-traces is an $\omega$-regular language.
- **Fact** (not shown in [PMC]): $\omega$-regular languages are closed under complementation.
- **Corollary**: The complement of $\mathsf{MinBadPref}(P) \cdot (2^{\mathsf{AP}})^{\omega}$, which is precisely $P$, is an $\omega$-regular language.

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- The preceding argument is non-constructive. But there is also a constructive argument for the same conclusion, *i.e.*, how to build an actual Büchi automaton that will recognize the regular safety property $P$.
  [PMC, Remark 4.31, page 177].

# verification of $\omega$-regular properties

- **Reminder:** A persistence property $P$ over AP is an LT property $P \subseteq (2^{\mathsf{AP}})^\omega$ if there is a propositional-logic formula $\Phi$ such that:

$$P = \left\{ A_0 A_1 A_2 \cdots \in (2^{\mathsf{AP}})^\omega \;\middle|\; \overset{\infty}{\forall} j. \text{ it holds that } A_j \models \Phi \right\}.$$

  The formula $\Phi$ is called the *persistence condition* (or *state condition*) of $P$. [PMC, Definition 4.61, page 199].

- **Fact:** Let the following be given:

  1. TS is a finite transition system over AP without terminal states.
  2. $P$ is an $\omega$-regular property over AP.
  3. $\mathscr{A}_B$ is an NBA over the alphabet $2^{\mathsf{AP}}$ that recognizes the complement of $P$, *i.e.*, $\mathscr{L}\left(\mathscr{A}_B\right) = (2^{\mathsf{AP}})^\omega - P$.

  Then we can define a state condition $\Phi$ s.t. the following are equivalent:

  (a) TS $\models P$
  (b) $\mathit{Traces}(\mathsf{TS}) \cap \mathscr{L}\left(\mathscr{A}_B\right) = \varnothing$
  (c) TS $\otimes \mathscr{A} \models P_\Phi$

  where $P_\Phi$ is the persistence property defined by the state condition $\Phi$. [PMC, Section 4.4, pages 198-201].

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