CS 512, Spring 2018, Handout 13

Algorithm for CTL Model-Checking

Assaf Kfoury

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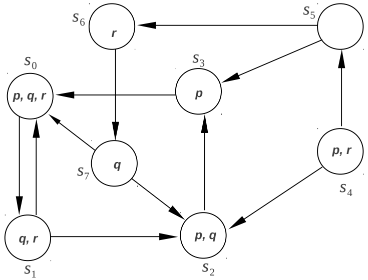
background reading for CTL model-checking algorithm

- [LCS, Section 3.6, pages 221-238] includes both:
  - Algorithm for CTL model-checking in [LCS, Subsections 3.6.1-2] ,
  - Algorithm for LTL model-checking in [LCS, Subsection 3.6.3] .
  - Useful exercise: Compare the two algorithms.
- [PMC, Section 6.4, pages 341-358] is another presentation of the same algorithm for CTL model-checking , with many more examples and explanations.
- Algorithm for CTL model-checking processes a given CTL WFF φ "from the inside out", *i.e.*, it starts from the smallest sub-WWF's and works outwards towards φ.
- In next slides, we illustrate the algorithm for CTL model-checking on an example, a variation of [PMC, Examples 6.26, 6.27, 6.28, pages 349, 353, 354] .

an example using CTL model-checking algorithm

TWO INPUTS:

1. $\varphi \triangleq \forall\bigcirc\ \exists\Diamond\ \psi$
   where
   $\psi \triangleq (\neg p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r)$
2. transition system TS (on the right, ***start states*** not specified)



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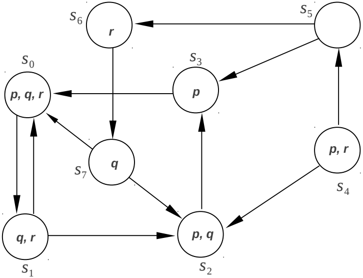
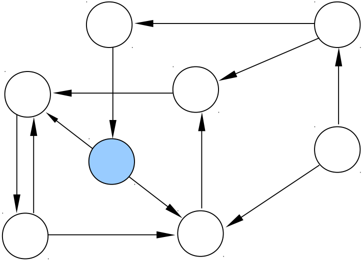


blue states are where sub-WFF: $(\neg p \wedge q \wedge \neg r)$ is satisfied



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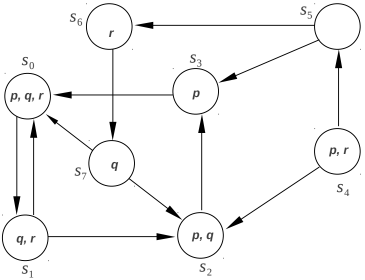
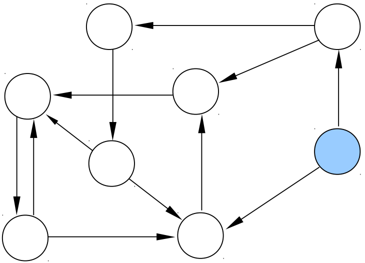


blue states are where sub-WFF: $(p \wedge \neg q \wedge r)$ is satisfied



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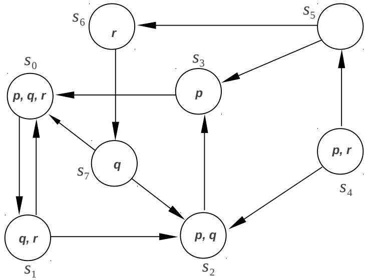

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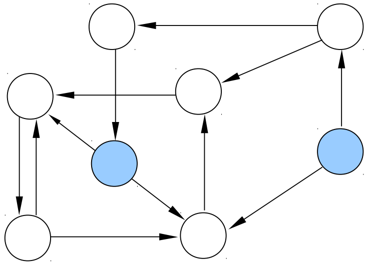

an example using CTL model-checking algorithm

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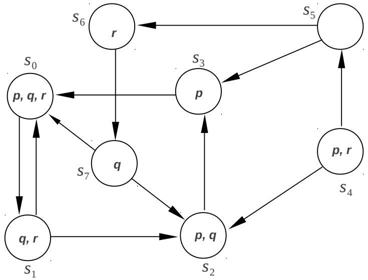
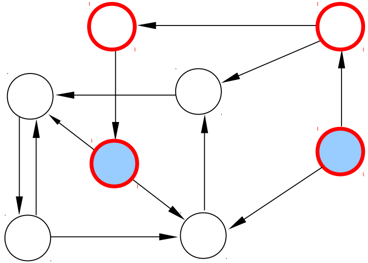


red states are where sub-WFF: $(\exists\Diamond\ \psi)$ is satisfied



an example using CTL model-checking algorithm

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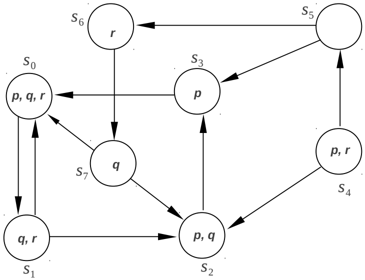
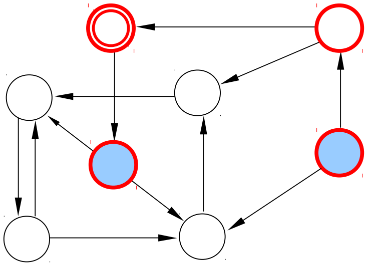


double-red states are where sub-WFF: $\forall\bigcirc\ (\exists\Diamond\ \psi)$ is satisfied

- **Conclusion:** TS $\models \varphi$ iff s_6 is one of the start states of TS



more on CTL model-checking algorithm

1. **Exercise**: Determine the states of TS (as defined in the preceding pages) satisfying the following CTL WFF's:

$$\begin{aligned} \varphi_1 &\triangleq \exists\Box\, q \\ \varphi_2 &\triangleq \forall\,(p \ \mathbb{U}\ \varphi_1) \ = \ \forall\,(p \ \mathbb{U}\ (\exists\Box\, q)) \\ \varphi_3 &\triangleq \exists\bigcirc\, \varphi_2 \ = \ \exists\bigcirc\,(\forall\,(p \ \mathbb{U}\ (\exists\Box\, q))) \end{aligned}$$

2. **Problem** – illustrating *abstraction* to alleviate ***state-explosion***:
For an arbitrary CTL WFF φ and an arbitrary transition system TS, let TS$[\varphi]$ be the transition system obtained as follows:

   - For every state s, if TS$, s \not\models \varphi$, then delete state s and all transitions (*i.e.*, edges) to s and all transitions from s.

   Prove the following statement:

   - For a state s in transition system TS and CTL WFF φ, we have that TS$, s \models \exists\Box\, \varphi$ iff two conditions:
     (i) TS$, s \models \varphi$ and
     (ii) there is a strongly-connected component of TS$[\varphi]$ with at least one transition (*i.e.*, edge) which is reachable from s.

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