CS 512, Spring 2018, Handout 15

Model-Checking: Comparison of LTL, CTL and CTL*

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overview

- **LTL vs. CTL:** The expressiveness of LTL and CTL can be compared by analyzing the properties of transition systems which can be formulated by one of the two logics but not by the other.
- **LTL vs. CTL:** It turns out that neither logic subsumes the other. LTL is incapable of expressing possibility properties, while CTL cannot express fairness properties.
- **LTL vs. CTL:** For particular examples,

  there is no CTL formula which is equivalent to the LTL formula $\varphi \triangleq \Diamond\Box\, a$.

  there is no LTL formula which is equivalent to the CTL formula $\Phi \triangleq \forall\Box\exists\Diamond\, a$.

overview

- **CTL vs. CTL*:** The lack of expressiveness of CTL, when compared to CTL*, is due to the requirement that the path quantifiers $\{\exists, \forall\}$ and the temporal operators $\{\bigcirc, \Diamond, \Box, \mathbb{U}\}$ alternate.

  All different ways of inserting quantifers in LTL wff $\varphi \triangleq \Diamond\Box\, a$ to obtain a CTL wff:

  $\underset{\uparrow}{\forall}\Diamond\Box\, a$ $\quad$ $\underset{\uparrow}{\exists}\Diamond\Box\, a$ $\quad$ (in first position only, legal in CTL* but not in CTL)

  $\Diamond\underset{\uparrow}{\forall}\Box\, a$ $\quad$ $\Diamond\underset{\uparrow}{\exists}\Box\, a$ $\quad$ (in second position only, legal in CTL* but not in CTL)

  $\underset{\uparrow}{\forall}\Diamond\underset{\uparrow}{\forall}\Box\, a$ $\quad$ $\underset{\uparrow}{\forall}\Diamond\underset{\uparrow}{\exists}\Box\, a$ $\quad$ (in first + second positions, legal in both CTL* and CTL)

  $\underset{\uparrow}{\exists}\Diamond\underset{\uparrow}{\forall}\Box\, a$ $\quad$ $\underset{\uparrow}{\exists}\Diamond\underset{\uparrow}{\exists}\Box\, a$ $\quad$ (in first + second positions, legal in both CTL* and CTL)

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  All different ways of inserting quantifers in LTL wff $\varphi \triangleq \Diamond\Box\, a$ to obtain a CTL wff:

  $$\underset{\uparrow}{\forall}\Diamond\Box\, a \qquad \underset{\uparrow}{\exists}\Diamond\Box\, a \qquad \text{(in first position only, legal in CTL* but not in CTL)}$$

  $$\Diamond\underset{\uparrow}{\forall}\Box\, a \qquad \Diamond\underset{\uparrow}{\exists}\Box\, a \qquad \text{(in second position only, legal in CTL* but not in CTL)}$$

  $$\underset{\uparrow}{\forall}\Diamond\underset{\uparrow}{\forall}\Box\, a \qquad \underset{\uparrow}{\forall}\Diamond\underset{\uparrow}{\exists}\Box\, a \qquad \text{(in first + second positions, legal in both CTL* and CTL)}$$

  $$\underset{\uparrow}{\exists}\Diamond\underset{\uparrow}{\forall}\Box\, a \qquad \underset{\uparrow}{\exists}\Diamond\underset{\uparrow}{\exists}\Box\, a \qquad \text{(in first + second positions, legal in both CTL* and CTL)}$$

- By [PMC, Thm. 6.18, p. 335],

  for any CTL $\Phi \in \{\forall\Diamond\forall\Box\, a,\ \forall\Diamond\exists\Box\, a,\ \exists\Diamond\forall\Box\, a,\ \exists\Diamond\exists\Box\, a\}$ and LTL $\varphi \triangleq \Diamond\Box\, a$,

  (1) ***either*** $\Phi \equiv \varphi$,

  (2) ***or*** there does not exist any LTL formula which is equivalent to Φ.

  For this particular LTL φ it turns out that (2) holds, by [PMC, Thm 6.21, p. 337].

overview

- **CTL vs. CTL*:** The logic CTL* removes the restriction of alternating the path quantifiers $\{\exists, \forall\}$ and the temporal operators $\{\bigcirc, \Diamond, \Box, \mathbb{U}\}$.
- As a result, CTL* strictly subsumes both LTL and CTL.

  For example, the CTL* formula $\forall\Diamond\Box\varphi$ (which is not legal in CTL)

  is equivalent to the LTL $\varphi \triangleq \Diamond\Box\varphi$.

comparing LTL, CTL, and CTL*

- $\varphi_{1,\text{LTL}} \triangleq \Box\,\neg p$ and $\varphi_{1,\text{CTL}} \triangleq \forall\,\Box\,\neg p$
  express the same property "p never holds"

- $\varphi_{2,\text{LTL}} \triangleq \Box\,(p \to \Diamond\, q)$ and $\varphi_{2,\text{CTL}} \triangleq \forall\,\Box\,(p \to \forall\,\Diamond\, q)$
  express the same property "whenever p happens, q eventually happens"

comparing LTL, CTL, and CTL*

- useful fact to prove non-equivalences between LTL and CTL.

  FACT: Let TS and TS$'$ be transition systems such that **Paths**$(\text{TS}') \subseteq$ **Paths**(TS) – or **Traces**$(\text{TS}') \subseteq$ **Traces**(TS) – and let φ be a formula of **LTL**.

  If TS $\models \varphi$ then TS$' \models \varphi$.

  The preceding fact does **not** hold if φ is a formula of **CTL**.

- **Exercise**: Write a formula of CTL which is a counter-example showing that the preceding fact fails for CTL.

comparing LTL, CTL, and CTL*

- useful fact to prove non-equivalences between LTL and CTL.

  FACT: Let TS and TS$'$ be transition systems
  such that **Paths**(TS$'$) $\subseteq$ **Paths**(TS) – or **Traces**(TS$'$) $\subseteq$ **Traces**(TS) –
  and let φ be a formula of **LTL**.

  If TS $\models \varphi$ then TS$'$ $\models \varphi$.

  The preceding fact does **not** hold if φ is a formula of **CTL**.

- **Exercise**: Write a formula of CTL which is a counter-example showing that the preceding fact fails for CTL.

- $\varphi_{3,\text{LTL}} \triangleq \Diamond \bigcirc p$ is **not equivalent** to $\varphi_{3,\text{CTL}} \triangleq \forall \Diamond \forall \bigcirc p$
  $\varphi_{3,\text{CTL}}$ can distinguish between two transition systems which $\varphi_{3,\text{LTL}}$ cannot

comparing LTL, CTL, and CTL*

- useful fact to prove non-equivalences between LTL and CTL.

  FACT: Let TS and TS′ be transition systems
  such that **Paths**(TS′) ⊆ **Paths**(TS) – or **Traces**(TS′) ⊆ **Traces**(TS) –
  and let φ be a formula of **LTL**.

  If TS $\models \varphi$ then TS′ $\models \varphi$.

  The preceding fact does **not** hold if φ is a formula of **CTL**.

- <u>**Exercise**</u>: Write a formula of CTL which is a counter-example showing that the preceding fact fails for CTL.

- $\varphi_{3,\text{LTL}} \triangleq \Diamond \bigcirc p$ is **not equivalent** to $\varphi_{3,\text{CTL}} \triangleq \forall \Diamond \forall \bigcirc p$
  $\varphi_{3,\text{CTL}}$ can distinguish between two transition systems which $\varphi_{3,\text{LTL}}$ cannot

  stronger fact: $\varphi_{3,\text{CTL}}$ can distinguish between two transition systems which no LTL formula can

- $\varphi_{4,\text{LTL}} \triangleq \Diamond \Box p$ is **not equivalent** to $\varphi_{4,\text{CTL}} \triangleq \forall \Diamond \forall \Box p$
  $\varphi_{4,\text{LTL}}$ holds in a transition system where $\varphi_{4,\text{CTL}}$ does not

  stronger fact: $\varphi_{4,\text{LTL}}$ expresses a property which no CTL formula can

comparing LTL, CTL, and CTL*

- $\varphi_{5,\mathsf{LTL}} \triangleq \bigcirc\, p$ is **not equivalent** to $\varphi_{5,\mathsf{CTL}} \triangleq \exists \bigcirc\, p$

comparing LTL, CTL, and CTL*

- $\varphi_{5,\text{LTL}} \triangleq \bigcirc\, p$ is **not equivalent** to $\varphi_{5,\text{CTL}} \triangleq \exists \bigcirc\, p$

- **No** LTL formula and **no** CTL formula is equivalent to the CTL* formula $\psi \triangleq \exists \bigcirc\, p \,\wedge\, \forall \Diamond \Box p$

  Question: Why is ψ not a WFF in the syntax of CTL?

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