CS 512, Spring 2018, Handout 20 Hoare Logic (Continued)

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March 21, 2018

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r := x;

q := 0;

while  $y \leq r$ 

**do** 
$$r := r - y$$

q := q + 1 od

r := x;

q := 0;

while  $y \leq r$ 

 $do \quad r := r - y$ 

q := q + 1 od

$$\{x = r + y \cdot q \land r < y\}$$

r := x;

q := 0;

while  $y \leq r$ 

$$q := q + 1 \quad \text{od}$$
$$\{x = r + y \cdot q\}$$
$$\{x = r + y \cdot q \land r < y\}$$

do  $r \cdot - r - v$ 

(program computes  $r = x \mod y$  and  $q = x \dim y$ )

r := x;

q := 0;

while  $y \leq r$ 

$$do \quad r := r - y$$

$$\{x = r + y \cdot (q + 1)\}$$

$$q := q + 1 \quad od$$

$$\{x = r + y \cdot q\}$$

$$\{x = r + y \cdot q \land r < y\}$$

(assignment)

r := x;

q := 0;

while  $y \leq r$ 

$$\{x = r - y + y \cdot (q + 1)\}$$
 (assignment)  
**do**  $r := r - y$   
 $\{x = r + y \cdot (q + 1)\}$  (assignment)  
 $q := q + 1$  **od**  
 $\{x = r + y \cdot q\}$   
 $\{x = r + y \cdot q \land r < y\}$ 

(program computes  $r = x \mod y$  and  $q = x \dim y$ )

(implied) (assignment)

(assignment)

r := x;

q := 0;

while  $y \leq r$ 

$$\{x = r + y \cdot q\}$$

$$\{x = r - y + y \cdot (q + 1)\}$$
do  $r := r - y$ 

$$\{x = r + y \cdot (q + 1)\}$$
 $q := q + 1$  od
$$\{x = r + y \cdot q\}$$

$$\{x = r + y \cdot q \land r < y\}$$

(program computes  $r = x \mod y$  and  $q = x \dim y$ )

r := x;

q := 0;

(program computes  $r = x \mod y$  and  $q = x \dim y$ )

r := x;

q := 0; $\{x = r + y \cdot q\}$ (partial-while) while  $y \leq r$  $\{x = r + y \cdot q \land y \leq r\}$ (implied)  $\{x = r + y \cdot q\}$ (implied)  ${x = r - y + y \cdot (q + 1)}$ (assignment) **do** r := r - y $\{x = r + y \cdot (q+1)\}$ (assignment) q := q + 1 od  $\{x = r + y \cdot q\}$  $\{x = r + y \cdot q \land r < y\}$ 

(program computes  $r = x \mod y$  and  $q = x \dim y$ )

r := x:  $\{x = r\}$ (assignment) q := 0; $\{x = r + y \cdot q\}$ (partial-while) while  $y \leq r$  $\{x = r + y \cdot q \land y \leq r\}$ (implied)  $\{x = r + y \cdot q\}$ (implied)  ${x = r - y + y \cdot (q + 1)}$ (assignment) **do** r := r - y $\{x = r + y \cdot (q+1)\}$ (assignment) q := q + 1 od  $\{x = r + y \cdot q\}$  $\{x = r + y \cdot q \land r < y\}$ 

(program computes  $r = x \mod y$  and  $q = x \dim y$ )

$\{x = x\}$	(assignment)
r := x;	
$\{x = r\}$	(assignment)
q := 0;	
$\{x = r + y \cdot q\}$	(partial-while)
while $y \leq r$	
$\{x = r + y \cdot q \land y \leqslant r\}$	(implied)
$\{x = r + y \cdot q\}$	(implied)
$\{x = r - y + y \cdot (q+1)\}$	(assignment)
do  r := r - y	
$\{x = r + y \cdot (q+1)\}$	(assignment)
q:=q+1 od	
$\{x = r + y \cdot q\}$	
$\{x = r + y \cdot q \land r < y\}$	

$\{\top\}$	(implied)
$\{x = x\}$	(assignment)
r := x;	
$\{x = r\}$	(assignment)
q := 0;	
$\{x = r + y \cdot q\}$	(partial-while)
while $y \leq r$	
$\{x = r + y \cdot q \land y \leqslant r\}$	(implied)
$\{x = r + y \cdot q\}$	(implied)
$\{x = r - y + y \cdot (q+1)\}$	(assignment)
do  r := r - y	
$\{x = r + y \cdot (q+1)\}$	(assignment)
q:=q+1 od	
$\{x = r + y \cdot q\}$	
$\{x = r + y \cdot q \land r < y\}$	

# using proof rules for PCA's (from last page of Handout 19)

$\{\top\}$	(implied)
$\{1 = 0!\}$	(assignment)
y := 1;	
$\{y = 0!\}$	(assignment)
z := 0;	
$\{y = z!\}$	(partial-while)
while $z \neq x$	
$\{y = z! \land z \neq x\}$	(implied)
$\{y \cdot (z+1) = (z+1)!\}$	(assignment)
<b>do</b> $z := z + 1$	
$\{y \cdot z = z!\}$	(assignment)
y := y * z od	
$\{y = z!\}$	
$\{y = z! \land z = x\}$	(implied)
$\{y = x!\}$	

(I use logical variable v instead of  $E_0$ )

y := 1;

z := 0;

while  $z \neq x$ 

**do** z := z + 1

y := y \* z od

(I use logical variable v instead of  $E_0$ )

y := 1;

z := 0;

while  $z \neq x$ 

**do** z := z + 1

y := y \* z od

 ${y = x!}$ 

(I use logical variable v instead of  $E_0$ )

y := 1;

z := 0;

while  $z \neq x$ 

**do** z := z + 1

y := y \* z od

$$\{y = z! \land z = x\}$$
$$\{y = x!\}$$

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(implied)

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(I use logical variable v instead of  $E_0$ )

y := 1;

z := 0;

while  $z \neq x$ 

do z := z + 1 y := y \* z od  $\{y = z! \land 0 \leq x - z < v\}$   $\{y = z! \land z = x\}$  $\{y = x!\}$ 

(implied)

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(I use logical variable v instead of  $E_0$ )

y := 1;

z := 0;

while  $z \neq x$ 

do 
$$z := z + 1$$
  
 $\{y \cdot z = z! \land 0 \leq x - z < v\}$  (assignment)  
 $y := y * z$  od  
 $\{y = z! \land 0 \leq x - z < v\}$   
 $\{y = z! \land z = x\}$  (implied)  
 $\{y = x!\}$ 

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(I use logical variable v instead of  $E_0$ )

y := 1;

z := 0;

while  $z \neq x$ 

$$\{y \cdot (z+1) = (z+1)! \land 0 \leq x - (z+1) < v\}$$
 (assignment)  
**do**  $z := z+1$   

$$\{y \cdot z = z! \land 0 \leq x - z < v\}$$
 (assignment)  
 $y := y * z$  **od**  

$$\{y = z! \land 0 \leq x - z < v\}$$
  

$$\{y = z! \land 0 \leq x - z < v\}$$
 (implied)  

$$\{y = x!\}$$

(I use logical variable v instead of  $E_0$ )

y := 1;

z := 0;

while 
$$z \neq x$$
  

$$\{y = z! \land z \neq x \land 0 \leqslant x - z = v\}$$
 (implied)  

$$\{y \cdot (z+1) = (z+1)! \land 0 \leqslant x - (z+1) < v\}$$
 (assignment)  
do  $z := z+1$   

$$\{y \cdot z = z! \land 0 \leqslant x - z < v\}$$
 (assignment)  
 $y := y * z$  od  

$$\{y = z! \land 0 \leqslant x - z < v\}$$
  

$$\{y = z! \land z = x\}$$
 (implied)  

$$\{y = x!\}$$

(I use logical variable v instead of  $E_0$ )

$$y := 1;$$

$$z := 0;$$

$$\{y = z! \land 0 \leq x - z\}$$
(total-while)
while  $z \neq x$ 

$$\{y = z! \land z \neq x \land 0 \leq x - z = v\}$$
(implied)
$$\{y \cdot (z+1) = (z+1)! \land 0 \leq x - (z+1) < v\}$$
(assignment)
do
$$z := z+1$$

$$\{y \cdot z = z! \land 0 \leq x - z < v\}$$
(assignment)
$$y := y * z \text{ od}$$

$$\{y = z! \land 0 \leq x - z < v\}$$
(implied)
$$\{y = z! \land z = x\}$$
(implied)

(I use logical variable v instead of  $E_0$ )

$$y := 1;$$

$$\{y = 0! \land 0 \leq x - 0\}$$
(assignment)
$$z := 0;$$

$$\{y = z! \land 0 \leq x - z\}$$
(total-while)
while  $z \neq x$ 

$$\{y = z! \land z \neq x \land 0 \leq x - z = v\}$$
(implied)
$$\{y \cdot (z + 1) = (z + 1)! \land 0 \leq x - (z + 1) < v\}$$
(assignment)
do
$$z := z + 1$$

$$\{y \cdot z = z! \land 0 \leq x - z < v\}$$
(assignment)
$$y := y * z \text{ od}$$

$$\{y = z! \land 0 \leq x - z < v\}$$
(implied)
$$\{y = z! \land 0 \leq x - z < v\}$$
(implied)
$$\{y = z! \land 0 \leq x - z < v\}$$

(I use logical variable v instead of  $E_0$ )

$$\{1 = 0! \land 0 \leq x - 0\}$$
 (assignment)  

$$y := 1;$$
 (assignment)  

$$z := 0;$$
 (total-while)  
while  $z \neq x$  (total-while)  
while  $z \neq x$  ( $y = z! \land 0 \leq x - z = v$ } (implied)  

$$\{y \cdot (z+1) = (z+1)! \land 0 \leq x - (z+1) < v\}$$
 (assignment)  

$$do \quad z := z+1$$
 ( $y \cdot z = z! \land 0 \leq x - z < v$ } (assignment)  

$$y := y * z \quad od$$

$$\{y = z! \land 0 \leq x - z < v\}$$
 (assignment)  

$$y := y * z \quad od$$

$$\{y = z! \land 0 \leq x - z < v\}$$
 (implied)  

$$\{y = x! \land z = x\}$$
 (implied)

(I use logical variable v instead of  $E_0$ )

$$\{ x \ge 0 \}$$
(implied)  

$$\{1 = 0! \land 0 \le x - 0 \}$$
(assignment)  

$$y := 1;$$
(assignment)  

$$z := 0;$$
(assignment)  

$$z := 0;$$
(total-while)  
while  $z \ne x$   

$$\{y = z! \land 0 \le x - z \}$$
(total-while)  
while  $z \ne x$   

$$\{y = z! \land z \ne x \land 0 \le x - z = v \}$$
(implied)  

$$\{y \cdot (z+1) = (z+1)! \land 0 \le x - (z+1) < v \}$$
(assignment)  

$$do \quad z := z+1$$
(system of a standard or a stand

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