CS 512, Spring 2018, Handout 20

Hoare Logic (Continued)

Assaf Kfoury

March 21, 2018

using proof rules for PCA's (program computes $r = x \text{ mod } y$ and $q = x \text{ div } y$)

$r := x;$

$q := 0;$

$\textbf{while}\ y \leqslant r$

$\quad \textbf{do} \quad r := r - y$

$\quad\quad q := q + 1 \quad \textbf{od}$

using proof rules for PCA's (program computes $r = x \bmod y$ and $q = x \operatorname{div} y$)

$r := x;$

$q := 0;$

while $y \leqslant r$

do $r := r - y$

$q := q + 1$ **od**

$\{x = r + y \cdot q \;\wedge\; r < y\}$

using proof rules for PCA's (program computes $r = x \bmod y$ and $q = x \operatorname{div} y$)

$r := x;$

$q := 0;$

while $y \leqslant r$

do $r := r - y$

$q := q + 1$ **od**

$\{x = r + y \cdot q\}$

$\{x = r + y \cdot q \ \wedge \ r < y\}$

using proof rules for PCA's (program computes $r = x \bmod y$ and $q = x \operatorname{div} y$)

$r := x;$

$q := 0;$

while $y \leqslant r$

do $r := r - y$

$\{x = r + y \cdot (q+1)\}$ (assignment)

$q := q + 1$ **od**

$\{x = r + y \cdot q\}$

$\{x = r + y \cdot q \ \wedge \ r < y\}$

using proof rules for PCA's (program computes $r = x \bmod y$ and $q = x \text{ div } y$)

$r := x;$

$q := 0;$

while $y \leqslant r$

$\{x = r - y + y \cdot (q+1)\}$ (assignment)

do $r := r - y$

$\{x = r + y \cdot (q+1)\}$ (assignment)

$q := q + 1$ **od**

$\{x = r + y \cdot q\}$

$\{x = r + y \cdot q \ \wedge \ r < y\}$

using proof rules for PCA's (program computes $r = x \bmod y$ and $q = x \text{ div } y$)

$r := x;$

$q := 0;$

while $y \leqslant r$

$\{x = r + y \cdot q\}$ (implied)

$\{x = r - y + y \cdot (q+1)\}$ (assignment)

do $r := r - y$

$\{x = r + y \cdot (q+1)\}$ (assignment)

$q := q + 1$ **od**

$\{x = r + y \cdot q\}$

$\{x = r + y \cdot q \ \wedge \ r < y\}$

using proof rules for PCA's (program computes $r = x \bmod y$ and $q = x \text{ div } y$)

$r := x;$

$q := 0;$

while $y \leqslant r$

$\{x = r + y \cdot q \wedge y \leqslant r\}$ (implied)

$\{x = r + y \cdot q\}$ (implied)

$\{x = r - y + y \cdot (q + 1)\}$ (assignment)

do $r := r - y$

$\{x = r + y \cdot (q + 1)\}$ (assignment)

$q := q + 1$ **od**

$\{x = r + y \cdot q\}$

$\{x = r + y \cdot q \ \wedge \ r < y\}$

using proof rules for PCA's (program computes $r = x \bmod y$ and $q = x \operatorname{div} y$)

$r := x;$

$q := 0;$

$\{x = r + y \cdot q\}$ (partial-while)

while $y \leqslant r$

$\{x = r + y \cdot q \wedge y \leqslant r\}$ (implied)

$\{x = r + y \cdot q\}$ (implied)

$\{x = r - y + y \cdot (q + 1)\}$ (assignment)

do $r := r - y$

$\{x = r + y \cdot (q + 1)\}$ (assignment)

$q := q + 1$ **od**

$\{x = r + y \cdot q\}$

$\{x = r + y \cdot q \;\wedge\; r < y\}$

using proof rules for PCA's (program computes $r = x \bmod y$ and $q = x \text{ div } y$)

$r := x;$

$\{x = r\}$ (assignment)

$q := 0;$

$\{x = r + y \cdot q\}$ (partial-while)

while $y \leqslant r$

$\{x = r + y \cdot q \wedge y \leqslant r\}$ (implied)

$\{x = r + y \cdot q\}$ (implied)

$\{x = r - y + y \cdot (q + 1)\}$ (assignment)

do $r := r - y$

$\{x = r + y \cdot (q + 1)\}$ (assignment)

$q := q + 1$ **od**

$\{x = r + y \cdot q\}$

$\{x = r + y \cdot q \ \wedge \ r < y\}$

using proof rules for PCA's (program computes $r = x \bmod y$ and $q = x \operatorname{div} y$)

$\{x = x\}$	(assignment)
$r := x;$	
$\{x = r\}$	(assignment)
$q := 0;$	
$\{x = r + y \cdot q\}$	(partial-while)
while $y \leqslant r$	
$\quad \{x = r + y \cdot q \wedge y \leqslant r\}$	(implied)
$\quad \{x = r + y \cdot q\}$	(implied)
$\quad \{x = r - y + y \cdot (q + 1)\}$	(assignment)
do $\; r := r - y$	
$\quad \{x = r + y \cdot (q + 1)\}$	(assignment)
$\quad q := q + 1$ **od**	
$\quad \{x = r + y \cdot q\}$	
$\{x = r + y \cdot q \; \wedge \; r < y\}$	

using proof rules for PCA's (program computes $r = x \bmod y$ and $q = x \text{ div } y$)

$\{\top\}$	(implied)
$\{x = x\}$	(assignment)
$r := x;$	
$\{x = r\}$	(assignment)
$q := 0;$	
$\{x = r + y \cdot q\}$	(partial-while)
while $y \leqslant r$	
$\quad \{x = r + y \cdot q \wedge y \leqslant r\}$	(implied)
$\quad \{x = r + y \cdot q\}$	(implied)
$\quad \{x = r - y + y \cdot (q + 1)\}$	(assignment)
do $\; r := r - y$	
$\quad \{x = r + y \cdot (q + 1)\}$	(assignment)
$\quad q := q + 1$ **od**	
$\quad \{x = r + y \cdot q\}$	
$\{x = r + y \cdot q \; \wedge \; r < y\}$	

using proof rules for PCA's (from last page of Handout 19)

$\{\top\}$	(implied)
$\{1 = 0!\}$	(assignment)
$y := 1;$	
$\{y = 0!\}$	(assignment)
$z := 0;$	
$\{y = z!\}$	(partial-while)
while $z \neq x$	
$\quad\{y = z! \wedge z \neq x\}$	(implied)
$\quad\{y \cdot (z + 1) = (z + 1)!\}$	(assignment)
do $z := z + 1$	
$\quad\{y \cdot z = z!\}$	(assignment)
$\quad y := y * z$ **od**	
$\quad\{y = z!\}$	
$\{y = z! \wedge z = x\}$	(implied)
$\{y = x!\}$	

using proof rules for TCA's (I use logical variable v instead of E_0)

$y := 1;$

$z := 0;$

while $z \neq x$

do $z := z + 1$

$y := y * z$ **od**

using proof rules for TCA's (I use logical variable v instead of E_0)

$y := 1;$

$z := 0;$

$\textbf{while}\ z \neq x$

$\quad \textbf{do}\quad z := z + 1$

$\qquad y := y * z \quad \textbf{od}$

$\{y = x!\}$

using proof rules for TCA's (I use logical variable v instead of E_0)

$y := 1;$

$z := 0;$

while $z \neq x$

do $z := z + 1$

$y := y * z$ **od**

$\{y = z! \wedge z = x\}$ (implied)

$\{y = x!\}$

$y := 1;$

$z := 0;$

while $z \neq x$

do $z := z + 1$

$y := y * z$ **od**

$\{y = z! \wedge 0 \leqslant x - z < v\}$

$\{y = z! \wedge z = x\}$ (implied)

$\{y = x!\}$

using proof rules for TCA's

(I use logical variable v instead of E_0)

$y := 1;$

$z := 0;$

while $z \neq x$

do $z := z + 1$

$\{y \cdot z = z! \wedge 0 \leqslant x - z < v\}$ (assignment)

$y := y * z$ **od**

$\{y = z! \wedge 0 \leqslant x - z < v\}$

$\{y = z! \wedge z = x\}$ (implied)

$\{y = x!\}$

using proof rules for TCA's (I use logical variable v instead of E_0)

$y := 1;$

$z := 0;$

while $z \neq x$

$\{y \cdot (z+1) = (z+1)! \wedge 0 \leqslant x - (z+1) < v\}$ (assignment)

do $z := z + 1$

$\{y \cdot z = z! \wedge 0 \leqslant x - z < v\}$ (assignment)

$y := y * z$ **od**

$\{y = z! \wedge 0 \leqslant x - z < v\}$

$\{y = z! \wedge z = x\}$ (implied)

$\{y = x!\}$

using proof rules for TCA's

(I use logical variable v instead of E_0)

$y := 1;$

$z := 0;$

while $z \neq x$

$\{y = z! \wedge z \neq x \wedge 0 \leqslant x - z = v\}$ (implied)

$\{y \cdot (z+1) = (z+1)! \wedge 0 \leqslant x - (z+1) < v\}$ (assignment)

do $z := z + 1$

$\{y \cdot z = z! \wedge 0 \leqslant x - z < v\}$ (assignment)

$y := y * z$ **od**

$\{y = z! \wedge 0 \leqslant x - z < v\}$

$\{y = z! \wedge z = x\}$ (implied)

$\{y = x!\}$

using proof rules for TCA's (I use logical variable v instead of E_0)

$y := 1;$

$z := 0;$

$\{y = z! \wedge 0 \leqslant x - z\}$ (total-while)

$\textbf{while } z \neq x$

$\{y = z! \wedge z \neq x \wedge 0 \leqslant x - z = v\}$ (implied)

$\{y \cdot (z+1) = (z+1)! \wedge 0 \leqslant x - (z+1) < v\}$ (assignment)

$\textbf{do} \quad z := z + 1$

$\{y \cdot z = z! \wedge 0 \leqslant x - z < v\}$ (assignment)

$y := y * z \quad \textbf{od}$

$\{y = z! \wedge 0 \leqslant x - z < v\}$

$\{y = z! \wedge z = x\}$ (implied)

$\{y = x!\}$

using proof rules for TCA's

(I use logical variable v instead of E_0)

$y := 1;$

$\{y = 0! \wedge 0 \leqslant x - 0\}$ (assignment)

$z := 0;$

$\{y = z! \wedge 0 \leqslant x - z\}$ (total-while)

$\textbf{while } z \neq x$

$\quad \{y = z! \wedge z \neq x \wedge 0 \leqslant x - z = v\}$ (implied)

$\quad \{y \cdot (z+1) = (z+1)! \wedge 0 \leqslant x - (z+1) < v\}$ (assignment)

$\textbf{do} \quad z := z + 1$

$\quad \{y \cdot z = z! \wedge 0 \leqslant x - z < v\}$ (assignment)

$\quad y := y * z \quad \textbf{od}$

$\quad \{y = z! \wedge 0 \leqslant x - z < v\}$

$\{y = z! \wedge z = x\}$ (implied)

$\{y = x!\}$

using proof rules for TCA's

(I use logical variable v instead of E_0)

$\{1 = 0! \;\wedge 0 \leqslant x - 0\}$ (assignment)

$y := 1;$

$\{y = 0! \;\wedge 0 \leqslant x - 0\}$ (assignment)

$z := 0;$

$\{y = z! \;\wedge 0 \leqslant x - z\}$ (total-while)

while $z \neq x$

$\qquad \{y = z! \wedge z \neq x \;\wedge 0 \leqslant x - z = v\}$ (implied)

$\qquad \{y \cdot (z+1) = (z+1)! \;\wedge 0 \leqslant x - (z+1) < v\}$ (assignment)

do $\quad z := z + 1$

$\qquad \{y \cdot z = z! \;\wedge 0 \leqslant x - z < v\}$ (assignment)

$\qquad y := y * z$ **od**

$\qquad \{y = z! \;\wedge 0 \leqslant x - z < v\}$

$\{y = z! \wedge z = x\}$ (implied)

$\{y = x!\}$

using proof rules for TCA's

(I use logical variable v instead of E_0)

$\{ x \geqslant 0 \}$ (implied)

$\{1 = 0! \land 0 \leqslant x - 0\}$ (assignment)

$y := 1;$

$\{y = 0! \land 0 \leqslant x - 0\}$ (assignment)

$z := 0;$

$\{y = z! \land 0 \leqslant x - z\}$ (total-while)

while $z \neq x$

$\{y = z! \land z \neq x \land 0 \leqslant x - z = v\}$ (implied)

$\{y \cdot (z + 1) = (z + 1)! \land 0 \leqslant x - (z + 1) < v\}$ (assignment)

do $z := z + 1$

$\{y \cdot z = z! \land 0 \leqslant x - z < v\}$ (assignment)

$y := y * z$ **od**

$\{y = z! \land 0 \leqslant x - z < v\}$

$\{y = z! \land z = x\}$ (implied)

$\{y = x!\}$

(THIS PAGE INTENTIONALLY LEFT BLANK)