CS 512 Notes: Lecture 2

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1 Set Operations

Given an *alphabet* $\Sigma = \{A, B\}$. We can define different operations on Σ, including:

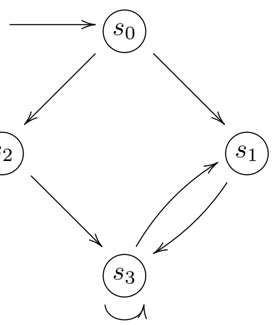
- *Concatenation*: $\{A, B\} \cdot \{A, B\} = \{AA, AB, BA, BB\}$.
  - We can use exponent notation as shorthand for multiple concatenations, as in $\{A, B\}^3 = \{AAA, \ldots, BBB\}$ (8 elements total).
- *Kleene star*: $\{A, B\}^* = \emptyset \cup \{A, B\} \cup \{A, B\}^2 \cup \ldots$.
- *Plus*: $\{A, B\}^+ = \{A, B\} \cup \{A, B\}^2 \cup \ldots$.
- *Omega*: If $\sigma \in \{A, B\}^\omega$ with $\sigma = x_0 x_1 x_2 x_3 \ldots$ (right-infinite), then it has $x_i \in \{A, B\}$ for each i.

1.1 Regular Expressions

The *language* of a regular expression can be expressed in terms of these set operations. For example, $\mathcal{L}((A + B)^*) = \Sigma^*$. Another example is $\mathcal{L}((A + B) \cdot (A + B) + A) = \Sigma^2 \cup \{A\} = \{AA, AB, BA, BB, A\}$.

2 Paths and Traces

Consider the following part of a transition system TS:



We can write down the infinite *paths* of TS using an *ω-regular expression*: $(s_0 s_1 + s_0 s_2) \cdot (s_3 + s_3 s_1)^\omega$. Now let AP (the atomic propositions) be $\{a, b\}$. Suppose we now add labels for observables. Label s_0 and s_1 as $\{a\}$, label s_2 as $\emptyset$, and label s_3 as $\{a, b\}$. Let $X = \emptyset$, $Y = \{a\}$, $Z = \{a, b\}$, and $W = \{b\}$. Then we can describe the infinite *traces* of TS using these labels: $(YY + YX) \cdot (Z + ZY)^\omega$. Note that the traces don't convey as much information as the paths, since both s_0 and s_1 have the same label.

3 Linear Time (LT) Properties

Continuing with our example from the previous section, we have $2^{\text{AP}} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\} = \{X, Y, W, Z\}$. The set $(2^{\text{AP}})^\omega$ consists of all infinite strings, where each part of the string comes from 2^{AP}. A *LT property* is $P \subseteq (2^{\text{AP}})^\omega$, a subset of the ω-traces of a transition system.

For example, let P_1 be "all infinite $\sigma = x_0 x_1 x_2 \cdots \in (2^{\text{AP}})^\omega$ such that for infinitely many $i \geq 0$ we have that $x_i = Y$." Using the symbol $\overset{\infty}{\exists}$ (there exists infinitely many), we can abbreviate the latter half of

P_1 as "$\overset{\infty}{\exists} i \geq 0$ with $x_i = Y$". Using a similar symbol $\overset{\infty}{\forall}$ (for all but finitely many, or *almost all*), we can create a second LT property $P_2 =$ "all infinite $\sigma = x_0 x_1 x_2 \cdots \in (2^{\mathrm{AP}})^\omega$ such that $\overset{\infty}{\forall} i \geq 0$, $x_i = Y$."

We can see that P_1 is satisfied if we loop $s_3 s_1$ infinitely many times. However, P_2 is not satisfiable since any infinite string visits s_3 (with label Z) infinitely many times. We write $TS \models P_1$, and $TS \not\models P_2$.

4 Finite Automata and Büchi Automata

See lecture slides for the notes.