CS 512 Notes: Lecture 4

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1 Homework 1 Problem 2 Solutions Correction

On the Homework 1 solution for problem 2, it was written that $closure(P_1) = P_1$. This, however, is not the case, the below example will show you why:

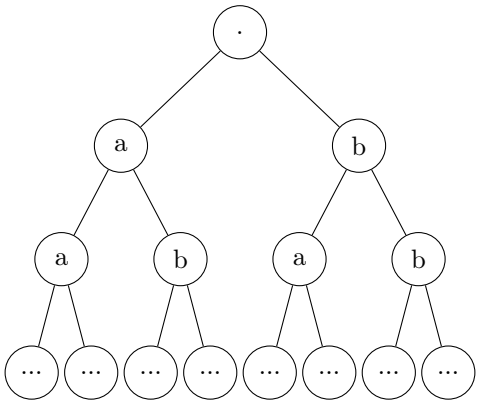
Say you have:

$$AP = \{a\}$$

which implies

$$2^{AP} = \{\phi, \{a\}\}$$

If we say $A = \phi$ and $B = \{a\}$, we can say that this powerset forms an alphabet $\Sigma = \{A, B\}$. This alphabet consists of words like $ABAAABAA$ (finite) and $ABABAB...$ (infinite). This can be visualized as a binary tree of the form:



The leftmost path is A^ω, rightmost is B^ω.

Let's define a property $P = \{A^m B^\omega | m \geq 0\}$. Then, since

$$pref(P) = \bigcup_{\sigma \in P} pref(\sigma)$$

It is also true that

$$pref(P) \supseteq \{\epsilon, A, AA, ..., A^n, ...\}$$

Informally, we can say that we take all prefixes and take them to their limit. Formally, we say

$$closure(P) = \{\sigma \in \Sigma^{omega} | pref(\sigma) \subseteq pref(P)\}$$

From the above, it should be clear that $closure(P) \neq P$ since $A^\omega \in closure(P)$ but $A^\omega \notin P$

A similar argument can be applied to Homework 1, problem 2 for why $closure(P_1) \neq P_1$.

2 Motivating Examples

2.1

Suppose

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P = {"x=0", "x > 1"}
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For example, these could be conditions in a piece of software. The powerset is: $2^{AP} = \emptyset, \{a\}, \{b\}, \{a, b\}$. For simplicity, we'll say $A = \emptyset, B = \{a\}, C = \{b\}, D = \{a, b\}$ (note that D is impossible).
Say we have a property with the follow conditions:

1. Initially, $x = 0$, so $P = \{A_0, A_1, ..., |A_0 = B\}$
2. $x > 1$ only finitely many times, so $P = \{A_0, A_1, ...|\exists i \geq 0, \forall j \geq i, b \notin A_j\}$. In other words, there is a step beyond which b is never the case.

2.2

Suppose you have some execution of a system where from some point on, the truth value of some $a \in AP$ alternates between true and false, or

$$P = \{A_0, A_1, ...|\exists i \geq 0, \forall j \geq i, a \in A_j iff a \notin A_{j+1}\}$$

This is a likeness property.