CS 512 Notes: Lecture 6

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6 February 2018

1 Motivating Example: Mutual Exclusion

$AP = \{wait, crit\}$ and the power set 2^{AP} of AP is $\{\emptyset, \{wait\}, \{crit\}, \{wait, crit\}\}$. The positive literal $crit$ stands for all sets $A \subseteq AP = \{wait, crit\}$ where the literal $crit$ holds, that is, the sets $\{crit\}$ and $\{wait, crit\}$. Let us assign M = $\emptyset$, N = $\{wait\}$, P = $\{crit\}$, Q = $\{wait, crit\}$.

1.1 ω-Regular Property

The property given by the informal statement "process $\mathcal{P}$ visits its critical section infinitely often" can be formalized by the following word :

$$P = \{A_0 A_1 ... A_i ... \in (2^{\mathrm{AP}})^\omega \mid \overset{\infty}{\exists} i.\ A_i = \{crit\} \text{ or } A_i = \{wait, crit\}\}$$

($\{wait, crit\}$ stands for "immediately enter into critical section")

The ω-regular expression of this word is :

$$((\mathrm{M}+\mathrm{N})^* \cdot (\mathrm{P}+\mathrm{Q}))^\omega$$

Since property $P = \mathcal{L}(((\mathrm{M}+\mathrm{N})^* \cdot (\mathrm{P}+\mathrm{Q}))^\omega)$, P is an ω-regular property.

1.2 Starvation Freedom Property

Starvation Freedom Property is a property that requires a process that wants to enter the critical section to be able to eventually do so. This property prevents a process from waiting ad infinitum. The property given by the informal statement "whenever $\mathcal{P}$ is waiting, then it will enter its critical section eventually later" means that the request will be accepted and enter its critical section after waiting some finite amount. This Starvation Freedom property is formalized by the set of words :

$$P = \{A_0 A_1 ... A_i ... \in (2^{\mathrm{AP}})^\omega \mid \forall i.\ if A_i = \{wait\} \text{ then } \exists\ j > i.\ A_j = \{crit\} \text{ or } A_j = \{wait, crit\}\}$$

$A_i = \{wait, crit\}$ is not in the precondition, for it is an extra condition. The ω-regular expression of this property is :

$$((\mathrm{M}+\mathrm{Q})^* \cdot \mathrm{N} \cdot \mathrm{M}^* \cdot (\mathrm{P}+\mathrm{Q})^+)^\omega + ((\mathrm{M}+\mathrm{Q})^* \cdot \mathrm{N} \cdot \mathrm{M}^* \cdot (\mathrm{P}+\mathrm{Q})^*)^* \cdot (\mathrm{M}+\mathrm{Q})^\omega$$

Intuitively, the first summand in the above expression stands for the case where $\mathcal{P}$ requests and enters its critical section infinitely often, while the second summand stands for the case where $\mathcal{P}$ is in its waiting phase only finitely many times.

2 Number of NBAs

The set $(2^{AP})^\omega$ is a collection of all sequences of ω - words, and all the entries come from 2^{AP}. And P is the property that can be exhibited (you are interested in). For example if the $|AP| = 2$, $\left|2^{AP}\right| = 4$, then $\left|(2^{AP})^\omega\right| = 4^\omega$. The size of real numbers $|\mathbb{R}| = \left|(2^{AP})^\omega\right| \ggg |\mathbb{N}|$(the size of natural numbers) .For every regular language, there is an NFA accepting it. Just like the English words in dictionary is not infinite, the number of regular expressions is equal to the number of natural numbers so as to the number of NFAs. NBA and NFA are the same objects but we use them on different ways. Thus, the number of possible NBAs is the number of natural numbers as well.