CS 512 Notes: Lecture 7

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1 Homework 2 Problem 1 clarification

There was some confusion abou that "$\rightarrow$ is the smallest relation defined by..."

First though, some clarifications: if $R \subseteq A \times A \times A$, then R is a ternary relation on A.

If we don't clarify what kind of relation R is, then it's fair to assume that R is actually a binary relation, i.e. $R \subseteq A \times B$.

Then, with respect to the homework question: If we say that R is a relation on A s.t. $(a, b) \in R$ and $(b, c) \in R$, then R is not uniquely defined. So when we say something "is the smallest relation defined by..." we're looking for a defintion that makes a uniquely defined relation.

2 LTL Equivalences

$\Phi \triangleq \Box(\mathit{Started} \rightarrow \mathit{Ready})$ means 'now and forever,' i.e., it's an invariant property.

$\Phi \triangleq \Box(\mathit{Requested} \rightarrow \Diamond \mathit{Acknowledged})$ means that if there is a request, it will eventually be acknowledged.

$\Phi \triangleq \Box\Diamond \mathit{Enabled}$ means for $A_0, A_1, A_2, ... \overset{\infty}{\exists} i \geq 0, \mathit{Enabled} \in A_i$

$\Phi \triangleq \Diamond\Box \mathit{Deadlocked}$ means after some point in the future, everything is deadlocked.

$\Phi \triangleq \Box\Diamond \mathit{Enabled} \rightarrow \Box\Diamond \mathit{Running}$ means if Enabled occurs infinitely often, then we are infinitely often running.

3 LTL Operators and Dualities

Primitive Ops from big book: $\circ$ means 'next' and $\Cup$ means 'until.'

3.1 Dualities

A duality between $\forall$ and $\exists$ implies that $\neg\forall\neg \equiv \exists$ and $\neg\exists\neg \equiv \forall$.

Some other dualities are:

$\vee$ and $\wedge$

$\Cup$ and R

3.2 Distributive property

$\Diamond(\Phi \wedge \Psi) \rightarrow \Diamond\Phi \wedge \Diamond\Psi$ is true BUT not the other way around (i.e., not iff).

Another way to phrase this is that $Words(\Diamond(\Phi \wedge \Psi)) \subseteq Words(\Diamond\Phi \wedge \Diamond\Psi)$

Also, note that $\Box(\Phi \vee \Psi) \not\equiv \Box\Phi \vee \Box\Psi$

With respect to the distributive property, $\Diamond$ behave like $\exists$ and $\Box$ behaves like $\forall$.