CS512 - Formal Methods

Thursday, February 22nd, 2018

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ASSIGNMENT #5 posted, due Wednesday, Feb. 28th.

<u>Definition:</u> Given a (state) formula Φ of CTL and a formula φ of LTL , we write $\Phi \equiv \varphi$ whenever for every transition system $TS : TS \vDash \Phi \longleftrightarrow TS \vDash \varphi$.

LTL , CTL can be under the umbrella of "**temporal logic**" but also, even more generally, "**model logic**".

There is the temptation to say "CTL is stronger than LTL , anything said in LTL can be said in CTL but not vice-versa." However, this isn't quite correct.

<u>Theorem:</u> 6.18 p335

Φ is a (state) formula of CTL .

φ is a formula of LTL s.t. φ obtained from Φ by omitting all-path ($\forall$) quantifiers.

Either

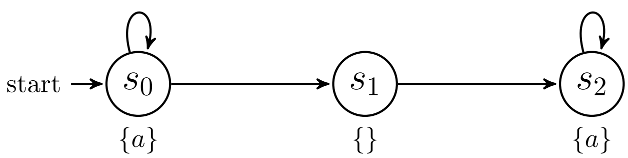
1. $\Phi \equiv \varphi$

   OR

2. there is no LTL formula that is equivalent to Φ

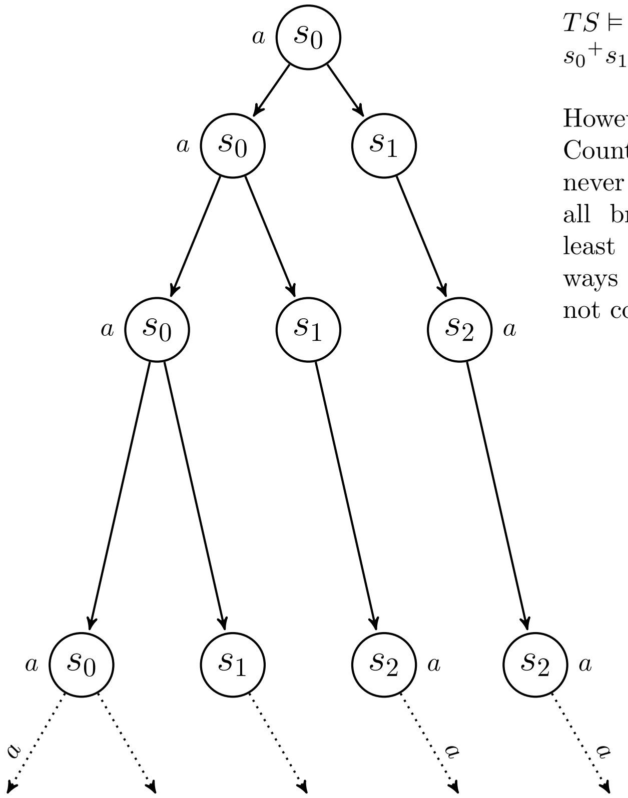
Case	Holds?	Φ CTL	φ LTL
1	✓	$\forall \bigcirc a$	$\bigcirc a$
1	✓	$\forall (a \mathbin{\mathsf{W}} b)$	$a \mathbin{\mathsf{W}} b$
1	✓	$\forall \Diamond a)$	$\Diamond a$
1	✓	$\forall \Box a$	$\Box a$
1	✓	$\forall \Box \forall \Diamond a$	$\Box \Diamond a$
2	No	$\forall \Diamond \forall \Box a$	$\Diamond \Box a$
2	No	$\forall \Diamond (a \wedge \forall \bigcirc a)$	$\Diamond (a \wedge \bigcirc a)$

<u>**Examining** $\forall \Diamond \forall \Box a$ **vs** $\Diamond \Box a$</u>



$\Phi \triangleq \forall \Diamond \forall \Box a$

$\varphi \triangleq \Diamond \Box a$



$TS \vDash \Diamond\Box a$
${s_0}^+ s_1 {s_2}^\omega$

However: $TS \nvDash \forall\Diamond\forall\Box a$
Counter-example: ${s_0}^*$ will never reach a point where all branches are $\Box a$, at least one branch will always have s_1 which does not contain a.

VIEWING HANDOUT 14
pg 3
$\forall x \;\; \varphi \triangleq \neg\exists x \neg\varphi$

Syntax is defined by *BNF* formula is now the standard. Furthermore, formal semantics are syntax-directed.

Note: $\neg\exists\neg\varphi$ is **NOT LEGAL** in *CTL*, but it **IS LEGAL** in *CTL**.

Handout 10 pg 11: syntax definition of *CTL* doesn't allow negation in path formulas φ.

Kripke - 1950's-60's: "modal logic"
Computer Science - 1980's-90's: "*CTL* , *LTL* , *CTL* *. It was eventually realized that these are extensions / redescriptions of Kripke's modal logic.

Handout 14, page 5

1. $TS, \pi \vDash \Phi \ldots$ why Φ, not φ? Because of the syntax!
   Recall: Path WFF: $\varphi ::= \Phi | \ldots$

Handout 14, page 7

LTL is a subset / sublogic of *CTL* *. *CTL* is a sublogic of *CTL* *.

Handout 15, page 2

Bullet #4: this is due to the way *LTL* , *CTL* syntax is defined. (See Handout 10).