CS 512 Notes: Lecture 13

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1 Handout 17 Material

1.1 HL Classical

HL Classical requires 3 things:

1. Syntax [expressions]
2. Formal Semantics (model theory)
3. Formal Proof/inference rules

2 and 3 are linked by soundness and completeness.

For LTL and CTL, we only talked about 1 and 2.

1.2 Semantics

Recall HL: $\{\phi\}P\{\psi\}$

The book uses something we call Operational Semantics. There are three aspects to this:

1. Small-step Operational Semantics
2. Big-step Operational Semantics
3. Reduction Operational Semantics

We use something known as Denotational Semantics in class, however.

For proofs, we use the following notation:
$\models$: Satisfaction. Can say $\models_{par}$ and $\models_{tot}$ for partial and total satisfaction.
$\vdash$: Derivability.

E.g. $\models_{par} \{\phi\}P\{\psi\}$

2 Handout 18 Materials

For assignment, we say $\{\psi[x \mapsto E]\}x := E\{\psi\}$.

Are the following rules sound?
Spec conjuction: Yes
Spec disjunction: Yes

These are not derivable from the rules in handout 17 and need official semantics

2.1 Formal Semantics (Denotational)

Note that we say $[[C]]$ is the formal semantics of C. $[[C]] : \Sigma \to \Sigma$ is a partial function of states. It's partial because it may diverge. $[[C]]_{rel} \subset \Sigma \times \Sigma$ can be interpreted from function.

$$\left.\begin{matrix}(\sigma, \sigma_1)\\(\sigma, \sigma_2)\end{matrix}\right\} \in R$$

which is how we get the relation.

For a slightly more complex example:

$[[x := E]]_{rel} \triangleq \{(\sigma, \sigma | x \mapsto n) | \sigma \in \Sigma \text{ and } n = [[E]]_\sigma\}$

$\sigma(x_1) = 3$
$\sigma(x_2) = -6$
$\sigma(x_3) = 20$
$...etc....$

Thus, $\sigma =< 3, -6, 20, ...? >$
Then we can also say $\sigma[x_2 \mapsto 21] =< 3, 21, 20, ... >$.

In our core language, we can picture the following lines of code as a graph:

```
while B do C od
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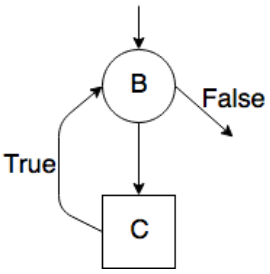


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if B then C; while B do C od; else skip
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