Practice Problem Set 2: Integrity (MACs & Signatures) Solutions

March 19, 2017

MAC security

The following is the security game for message authentication codes (MACs).

- The game master chooses a random k to the MAC.
- The adversary has access to a $MAC_k()$ oracle, that computes MACs on messages of the adversary's choice.
- The adversary has access to a $VER_k(,)$ oracle, that Verifies that a tag t is a valid MAC on a message m; both m and t can be chosen by the adversary.
- The adversary wins if outputs $m*, t*$ such that $m*$ has not been queried to the $MAC_k()$ oracle and $VER_k(m*, t*) = 1$.

We say the MAC is secure if no (polynomial time) adversary can win this game with probability better than about $\frac{1}{2^\ell}$, where ℓ is the length of the MAC tag.

Signature security

The following is the security game for digital signatures.

- The game master chooses a random asymmetric key (PK, SK) for the signature and gives PK to the adversary.
- The adversary has access to a $Sign_{SK}()$ oracle, that computes signatures on messages of the adversary's choice.
- The adversary wins if outputs $m*, \sigma*$ such that $m*$ has not been queried to the $Sign_{SK}()$ oracle and $VER_{PK}(m*, \sigma*) = 1$.

We say the digital signature is secure if no (polynomial time) adversary can win this game with non-negligible probability.

Questions.

Exercise 1. Show that

$$MD5(k||m)$$

is not a secure MAC. That is, present an attack that allows the adversary to win the MAC security game described above.

(Hint: Recall the length extension attack from Lab 1.)

Solution: Query the MAC oracle on message m to obtain $t = MD5(m||k)$. Now length extend that tag t as shown in Lab 1 to get $t^* = length\_extend(t, l)$ and $m* = m||l$. Now output $m*, t*$. You win the game because $m*, t*$ pass verification and $m*$ has never been queried to the MAC oracle.

Exercise 2. On February 23, 2017, researchers announced that they found a collision in SHA1. The collision was two files f_1 and f_2 such that $SHA1(f_1) = SHA1(f_2)$. See `shattered.io`.

Consider PKCS #1 v1.5 RSA digital signatures. To sign a message m, the message is hashed and padded as shown below to obtain the padded value $p(m)$:

$$\texttt{00}\quad\texttt{01}\quad\underbrace{\texttt{FF}\cdots\texttt{FF}}_{k/8-38\text{ bytes wide}}\quad\texttt{00}\ \underbrace{\texttt{3021300906052B0E03021A05000414}}_{\text{ASN.1 ``magic'' bytes}}\quad\underbrace{\texttt{XX}\cdots\texttt{XX}}_{\text{SHA1}(m)\text{ (20-bytes)}}$$

Then, the signature is

$$p(m)^d \mod N$$

where N is the RSA modulus, d is the secret RSA decryption exponent, and e is the public encryption exponent. Thus, the public key is (e, N) and the secret key is (d, N).

Present an attack that proves that PKCS #1 v1.5 RSA is not a secure digital signatures when SHA1 is used as the hash function. You must use the two files f_1 and f_2 in your attack.

Solution: Attacker queries signing oracle for $\sigma = Sign_{SK}(f_1)$ now he knows $\sigma = p(f_1)^d$. We know that f_0, f_1 hash to the same value and p just appends the hash with padding so, $p(f_0) = p(f_1)$ and therefore $p(f_1)^d = p(f_0)^d$. So to win the game attacker outputs f_0, σ and wins because they pass verification and f_0 has never been queried to the signing oracle.

Exercise 3. Dr Snakeoil markets a new product that he claims protects the integrity of messages.

This product requires Alice and Bob to share a secret key 128-bit key k that they will use to authenticate every message they send.

Then, if Alice wants to send a message m to Bob, she breaks the message m up into blocks $m_1, m_2, ..., m_n$ and outputs the tag $t_1, t_2, ..., t_i, ..., t_n$ where each $t_i = HMAC_k(m_i)$.

Alice then sends $m_1, m_2, ..., m_n, t_1, t_2, ..., t_n$ to Bob.

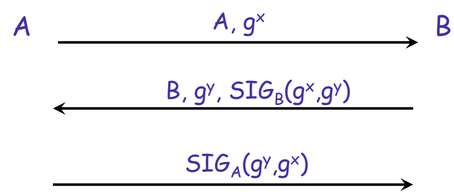
1. Write down the verification algorithm for this scheme.

   Solution: Verify that $t_1 = HMAC_k(m_1), \ldots, t_n = HMAC_k(m_n)$ for all $(t_1, m_1), \ldots, (t_n, m_n)$ pairs.

2. Prove that this scheme is not a secure MAC.

   Solution: Ask the MAC oracle for $t^a = t_1^a, \ldots, t_n^a = HMAC_k(m_1^a), \ldots, HMAC_k(m_n^a)$ for a message m^a. Then ask the MAC oracle for $t^b = t_1^b, \ldots, t_n^b = HMAC_k(m_1^b), \ldots, HMAC_k(m_n^b)$ for a message m^b. Now construct a new $t' = t_1^a, \ldots, t_{n/2}^a, t_{n/2+1}^b, \ldots, t_n^b$ and corresponding message $m' = n_1^a, \ldots, m_{n/2}^a, m_{n/2+1}^b, \ldots, m_n^b$. $Ver_k(m', t')$ is true, and you never queried the MAC oracle for m', so you break the security definition of a MAC.

Exercise 4. (Key exchange). Consider the following diffie-helman key-exchange protocol. Recall that the shared key is $k = g^{xy}$, and that $SIG_A(m)$ is the (public-key) digital signature on message m signed by the secret key of A. Suppose that A, B and E all know each other's correct public keys.



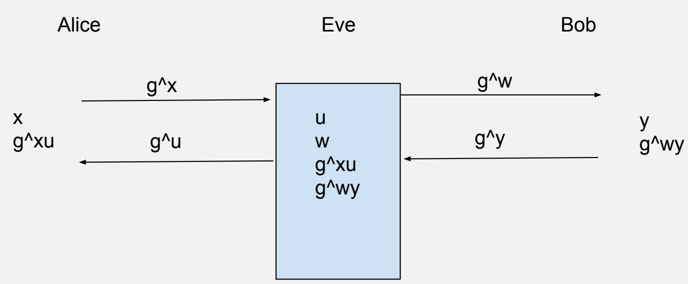
After this protocol runs, Alice and Bob send each other messages encrypted and authenticated under the key k.

Suppose there is a man-in-the-middle adversary E that can intercept, add, drop, and the modify the traffic that A sends to B.

1. Suppose that Alice and Bob are running software that has the following implementation flaw: it forgets to validate digital signatures and just accepts any messages it receives as valid.

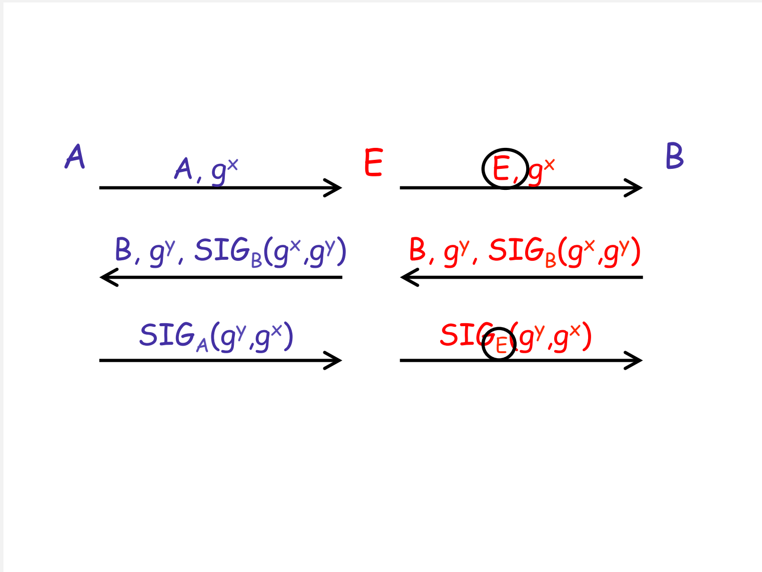
   Show how Eve E can launch an man-in-the-middle attack, where she can read any of the encrypted and authenticated messages that Alice sends Bob.

   Solution: Eve generates w and u. She then can create a key that she shares with Bob g^{wy} and a key that she shares with Alice g^{xu}. With these two keys, Eve can decrypt messages from Alice using the key g^{xu} and re-encrypt messages before she sends them to Bob with g^{wy}. I've left out the signatures from the following diagram since they are not checked.

2. Now suppose E can launch an "identity misbinding attack" where she convinces B that he shares the key $k = g^{xy}$ with E, while convincing A that she shares $k = g^{xy}$ with B. Explain exactly how E does this. (What messages does she send, and to who?) [Note, with this attack, E doesn't know $k = g^{xy}$ but B considers anything sent by A as coming from E]

Solution: Eve initially send Bob her identity E. Bob now thinks that he's getting messages from Eve but really he's getting messages from Alice. Thus the identity is "misbinded".



3. Give an example of a scenario where your identity misbinding attack might create problems.

Solution: Alice asks Bob "put $100 in my account". Bob thinks he's communicating with Eve so he puts $100 into Eve's account.